PHYSICS 315
Numerical Data

Speed of light	c = 3 x 10^{10} cm s^{-1}
Mass of the Hydrogen atom	m_H = 1.67 x 10^{-24} g
Mass of the Helium atom	m_{He} = 3.97 m_H
Size of a nucleon	$\approx 10^{-13}$ cm
Mass of the electron	m_e = 9.11 x 10^{-28} g
Charge of electron	e = 4.80 x 10^{-10} e.s.u.
Gravitational constant	G = 6.67 x 10^{-8} dyne cm^2 g^{-2}
Stefan-Boltzmann constant	σ = 5.67 x 10^{-5} erg cm^{-2} deg^{-4} s^{-1}
Radiation constant	a = 7.566 x 10^{-15} erg cm^{-3} deg^{-4}
Planck constant	h = 6.63 x 10^{-27} erg s
Boltzmann constant	k = 1.38 x 10^{-16} erg deg^{-1}
Energy of 1 eV	= 1.60 x 10^{-12} erg
Deceleration parameter	q_0 = -0.55
Angstrom	1$\mathring{A}$ = 10^{-8} cm
Bohr radius	$a_0 = 5.3 \times 10^{-9}$ cm
Parsec	1 pc = 3.086 x 10^{18} cm
Mean earth-sun distance	1 AU = 1.496 x 10^{13} cm
Solar radius	$R_\odot$ = 6.96 x 10^{10} cm
Solar mass	$M_\odot$ = 1.99 x 10^{33} g
Solar luminosity	$L_\odot$= 3.85 x 10^{33} erg s^{-1}
Effective temperature of Sun	$T_\odot$ = 5781 K
Absolute magnitude of the Sun	M_V = +4.8
Solar abundance	X = 0.707, Y = 0.274, Z = 0.0189
Thomson scattering cross-section	$\sigma_T = 6.65 \times 10^{-25}$ cm^2
1 Jansky	1 Jy = 10^{-23}erg s^{-1} cm^{-2} Hz^{-1}
# ergs/Joule	= 10^7
# arcseconds/radian	= 206,265
# of seconds/year	= 3.156 x 10^7
Typical seeing	1″
Recombination coefficient, α_r	$= 3.1 \times 10^{-13}$ $cm^3\,s^{-1}$
Photon energy required to ionize hydrogen	= 13.6 eV
Hα, Hβ wavelengths (respectively)	= 656 nm, 488 nm
HI λ21 cm wavelength	= 21.106 cm
$A_{j,i}$(21 cm line)	$= 2.876 \times 10^{-15}$ s^{-1}
γ(21 cm line)	$= 9.5 \times 10^{-11}$ $cm^3\,s^{-1}$
Lyα wavelength	= 121.567 nm
Ionization frequency of hydrogen	$= 3.29 \times 10^{15}$ Hz

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Equations (cgs units assumed unless otherwise indicated)

$$\mathcal{F} = ma = m\frac{d^2x}{dt^2} \qquad \mathcal{F} = \frac{Gm_1m_2}{r^2} \qquad a = \frac{v^2}{r}$$

$$\mathcal{F}_{rad} = \frac{2f}{c} A\,cos^2\theta \qquad \mathcal{F}_{rad} = \frac{f}{c} A\,cos\theta \qquad \theta_{res} \sim \lambda/D$$

$$E = mc^2 \qquad \lambda = h/p \qquad E = h\,\nu \qquad \lambda\,\nu = c \qquad \tfrac{\Delta\lambda}{\lambda_0} = \tfrac{v}{c} \qquad c\,z = H_o\,D$$

$$dE = I_\nu \cos\theta\, d\Omega\, d\nu\, dA\, dt \quad f_\nu = \int I_\nu cos\theta d\Omega \quad L = \int F\, dA \quad d\Omega = sin\theta d\theta d\phi$$

$$f = F\,(\tfrac{R}{r})^2 \qquad F = \pi\,\mathrm{I} \;\; \text{(stellar surface)} \qquad \Omega = \tfrac{\pi\theta^2}{4}$$

$$u = \tfrac{4\pi}{c}\,J \qquad \mathcal{P}_{rad} = \tfrac{1}{3}\,u \qquad \mathcal{P}_{particles} = n\,k\,T = \tfrac{\rho}{\mu\, m_H}\,k\,T \qquad \mu \equiv \tfrac{<m>}{m_H}$$

$$\mu = \tfrac{1}{2X + 3Y/4 + Z/2} \qquad \text{ionized}$$

$$\text{Black Body}: \quad B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1} \qquad B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{(e^{h\nu/kT} - 1)}$$

$$B_\lambda(T) = \frac{2\,c\,k\,T}{\lambda^4} \qquad B_\nu(T) = \frac{2\,k\,T\,\nu^2}{c^2} \qquad B_\lambda(T) = \frac{2\,h\,c^2}{\lambda^5}\,e^{-(\frac{h\,c}{\lambda\,k\,T})} \qquad B_\nu(T) = \frac{2\,h\,\nu^3}{c^2}\,e^{-(\frac{h\,\nu}{k\,T})}$$

$$\lambda_{\max}T = 0.29 \qquad F = \sigma T^4 \qquad u = a\,T^4 \qquad N_i\;[\text{photons/s}] = L_i/(h\nu)$$

$$\ell = \frac{1}{\kappa\,\rho} = \frac{1}{\sigma\,n} = \frac{1}{\alpha} \qquad \tau_\nu = \int -\kappa_\nu\,\rho\,dr = \int -\alpha_\nu\,dr$$

$$\gamma = v\,\sigma \quad t = \frac{1}{n\,v\,\sigma} \qquad \text{collision rate per unit volume: } \nu = n_1\,n_2\,\gamma$$

$$\frac{d\mathrm{I}_\nu}{dr} = -\alpha_\nu\,\mathrm{I}_\nu + j_\nu \qquad \frac{d\mathrm{I}_\nu}{d\tau_\nu} = \mathrm{I}_\nu - S_\nu \qquad S_\nu = \tfrac{j_\nu}{\alpha_\nu}$$

$$\mathrm{I}_\nu = I_{\nu_0}e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu}) \qquad \mathrm{I}_\nu = \mathrm{I}_{\nu_0}e^{-\tau_\nu} + B_\nu(T)(1 - e^{-\tau_\nu}) \quad \text{(LTE)}$$

$$\mathrm{T}_{B_\nu} = \mathrm{T}_{B_{\nu_0}}e^{-\tau_\nu} + T\,(1 - e^{-\tau_\nu}) \quad (\text{LTE}, \;\; h\,\nu << k\,\mathrm{T})$$

$$T_{eq} = \left[\tfrac{(1-A)L_\odot}{16\pi\sigma r^2}\right]^{1/4} \qquad \text{Kepler's 3rd law: } \left[\tfrac{P}{\mathrm{yr}}\right]^2 = \left[\tfrac{a}{AU}\right]^3$$

$$U = R_s\,n_e^{2/3} = (\tfrac{3N_*}{4\pi\alpha_r})^{1/3} \qquad <v> = \sqrt{\tfrac{8\,k\,\mathrm{T}}{m}} \qquad v_{mp} = \sqrt{\tfrac{2\,k\,\mathrm{T}}{m}} \qquad d < c\,\Delta\,t$$

Thermal Bremsstrahlung :

$$j_\nu \;=\; 5.44\;\times\;10^{-39}\,(\frac{Z^2}{T_e^{1/2}})\,n_i\,n_e\,g(\nu,T_e)\,e^{-\left(\frac{h\,\nu}{k\,T_e}\right)}$$

$$g(\nu,T_e)\;=\;11.962\,T_e^{0.15}\,\nu^{-0.1}\qquad\text{(radio)}$$

$$\left[\frac{n_e}{cm^{-3}}\right]\;=\;(1.55\;\times\;10^{-19}\,f_X)\,\left[\frac{L_X}{erg\,s^{-1}}\right]^{0.5}\,\left[\frac{V}{kpc^3}\right]^{-0.5}\,\left[\frac{T_e}{K}\right]^{-0.25}$$

where $$f_X\;=\;\left[g_X\left(e^{-\frac{E_1}{kT_e}}\;-\;e^{-\frac{E_2}{kT_e}}\right)\right]^{-0.5}\qquad\text{(X - ray)}$$

Synchrotron :

$$j_\nu\;=c_5(\Gamma)\,N_0\,B_\perp^{\frac{(\Gamma+1)}{2}}\,(\frac{\nu}{2\,c_1})^{\frac{(1-\Gamma)}{2}}\qquad\alpha_\nu\;=c_6(\Gamma)\,N_0\,B_\perp^{\frac{(\Gamma+2)}{2}}\,(\frac{\nu}{2\,c_1})^{\frac{-(\Gamma+4)}{2}}$$

$$I_\nu\;=\;f(\Gamma)\,B_\perp^{-1/2}\,\nu^{5/2}\,(1\;-\;e^{-\tau_\nu})\qquad[\text{where }f(\Gamma)\;\equiv\;c_5(\Gamma)/c_6(\Gamma)]$$

$$r\;=\;1706.5\,\frac{E}{m_e\,c^2}\,\frac{1}{B}\qquad\alpha\;=\;-(\Gamma-1)/2\qquad n(E)\;=\;N_0\,E^{-\Gamma}\;=>\;n_{CRe}\;=\;\int N_0\,E^{-\Gamma}dE$$

$$\tau\;=\;c_6(\Gamma)\,N_0\,B_\perp^{\frac{\Gamma+2}{2}}\,\left(\frac{\nu}{2c_1}\right)^{-\frac{(\Gamma+4)}{2}}\;l$$

Lines :

$$\Delta E\;=\;E_i-E_j=(\frac{1}{j^2}-\frac{1}{i^2})(\frac{m_e e^4}{2\hbar^2})=h\nu\qquad\Delta v\;=\;2.15\;\times\;10^4\,(\frac{T}{\mathcal{A}})^{1/2}$$

$$\frac{N_{II}}{N_I}=\frac{1}{n_e}[\frac{2\pi m_e kT}{h^2}]^{3/2}\;e^{-\chi_i/kT}\qquad\frac{N_i}{N_j}\;=\;\frac{g_i}{g_j}e^{-(E_i-E_j)/kT}\qquad g_i\;=\;2\,i^2$$

$$\Phi_D(v)\;=\;\frac{1}{\sqrt{\pi}\,b}\,exp\left(\frac{-v^2}{b^2}\right)\qquad b\;=\;\sqrt{\left(\frac{2\,k\,T}{m}\right)}$$

$$\mathcal{N}_{HI}\;=\;1.823\;\times\;10^{18}\,\int T_B(HI_{21cm})\,dv\quad cm^{-2}\;\;(v\text{ in }km\,s^{-1})$$

$$\mathcal{N}_{H_2}\;=\;2.3\;\times\;10^{20}\,\int T_B\,[CO(J=1-0)]\,dv\quad cm^{-2}\;\;(v\text{ in }km\,s^{-1})$$

$$M_{HI}\;=\;2.35\;\times\;10^5\,D^2\int f_\nu\,dv\qquad(f_\nu\text{ in }Jy,\;v\text{ in }km\,s^{-1},\;D\text{ in }Mpc,\;M_{HI}\text{ in }M_\odot)$$

Cosmology

$$H_0\,D_L\;=\;c\,z\left[1\;+\;\frac{1}{2}\,(1\;-\;q_0)\;z\right]\qquad z\;<\;1$$

The Electromagnetic Spectrum

Gamma Ray: $\lambda < 0.01$ nm
X-ray: 0.01 nm $< \lambda <$ 10 nm
UV: 10 nm $< \lambda <$ 400 nm
Visible: 400 nm $< \lambda <$ 700 nm
IR: 700 nm $< \lambda <$ 1 mm
Microwave: 1 mm $< \lambda <$ 10 cm
Radio[1]: $\lambda > 10$ cm

[1]The radio regime is sometimes taken to include the microwave.