PHYSICS 372: Numerical Data & Unit Conversions

Mass of an atomic mass unit	$m_{amu} = 1.67 \times 10^{-27}$ kg
Mass of the electron	$m_e = 9.11 \times 10^{-31}$kg
Planck's constant	h = 6.626×10^{-34} J s
Boltzmann's constant	k = 1.38×10^{-23} J K^{-1}
Speed of light	c = 3×10^8 m s^{-1}
Angstrom	$1\mathring{A} = 10^{-10}$ m
Avogadro's number	$N_A = 6.02 \times 10^{23}$
Gas constant	R = 8.315 J/(mol K)
dry air mean molecular weight	$\mu = 29$

T (in °C) = T(in K) - 273

1 atm = 1.013 bar = 1.013×10^5 N/m^2

PHYSICS 372: Mathematical Expressions

$$f = f(x,y) \quad => \quad df = \frac{\partial f}{\partial x}\Big|_y dx + \frac{\partial f}{\partial y}\Big|_x dy$$

Stirling's Approx: $N! \approx N^N e^{-N}\sqrt{2\pi N}$ $\qquad ln(N!) \approx NlnN - N \quad$ (N large)

$ln(1 + x) \approx x \qquad (1 + ax)^{1/x} \approx e^a \qquad (|x| << 1)$

Cyclic relation for P, V, & T: $\left(\frac{\partial V}{\partial P}\Big|_T \left(\frac{\partial P}{\partial T}\Big|_V \left(\frac{\partial T}{\partial V}\Big|_P = -1\right.\right.\right.$

Mixed partial derivatives: $\frac{\partial^2 f}{\partial x\,\partial y} = \frac{\partial^2 f}{\partial y\,\partial x}$

PHYSICS 372: Equations

Ideal gas law: $\qquad PV = NkT = nRT$

Ideal gas *rms* speed: $v_{rms} = \sqrt{\frac{3kT}{\mu m_{amu}}}$ where $\mu \equiv \frac{<m>}{m_{amu}}$ is the mean mol. wt.

Van der Waal's gas: $\qquad \left(P + \frac{a}{v^2}\right)(v - b) = RT \qquad$ where $\qquad v = V/m$

$\gamma = \frac{f+2}{f}$ $\qquad$ Quadratic degrees of freedom: $U = N\frac{f}{2}kT$

Volume expansion coefficient: $\beta \equiv \frac{1}{V}\left(\frac{\partial V}{\partial T}\Big|_P\right.$ Isothermal compressability: $\kappa \equiv -\frac{1}{V}\left(\frac{\partial V}{\partial P}\Big|_T\right.$

Sound speed: $c_s = \sqrt{\frac{B_s}{\rho}}$ where $\frac{B_s}{\gamma} = B = \frac{1}{\kappa} = \frac{1}{\gamma\,\kappa_s}$ (subscripts, s, meaning adiabatic)

First law: $\Delta U = Q + W \qquad dU = dQ + dW$

Expansive/compressive work: $dW = -P\,dV$

Ideal gas adiabatic expansion/compression: $V\,T^{f/2} = const \qquad P\,V^{\gamma} = const$

Enthalpy: $H = U + PV \qquad$ Latent heat: $L = \frac{Q}{m}$

Heat Capacities: $C_V \equiv \left(\frac{dQ}{dT}\right|_V = \left(\frac{\partial U}{\partial T}\right|_V$

$$C_P \equiv \left(\frac{dQ}{dT}\right|_P = \left(\frac{\partial H}{\partial T}\right|_P = C_V + \left[\left(\frac{dU}{dV}\right|_T + P\right] V\beta = C_V + \frac{\beta^2 TV}{\kappa}$$

Two-state system: $\quad \Omega(N,n) = \dfrac{N!}{n!(N-n)!} \qquad (N > n)$

Einstein solid: $\quad \Omega(N,q) = \frac{(q+N-1)!}{q!(N-1)!} \qquad$ (N the # of oscillators, $q\,\epsilon = U$)

$$\Omega = \left(\frac{e\,N}{q}\right)^q \quad (q << N), \qquad \Omega = \left(\frac{e\,q}{N}\right)^N \quad (q >> N)$$

Monatomic ideal gas: $\Omega = f(N)\,V^N\,U^{3N/2}$

Entropy: $S = k\,ln\Omega$

Sackur-Tetrode Eqn: $\quad S = Nk\left\{ln\left[\frac{V}{N}\left(\frac{4\pi mU}{3Nh^2}\right)^{3/2}\right] + \frac{5}{2}\right\}$

$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right|_{N,V} \qquad dS = \frac{dQ}{T} \qquad dS = \frac{C_V}{T}\,dT$ (V const), or $dS = \frac{C_P}{T}\,dT$ (P const)

TdS Equations:

$$T\,dS = C_V\,dT + T\,\frac{\beta}{\kappa}\,dV$$

$$T\,dS = C_P\,dT - \beta VT\,dP$$

$$T\,dS = \frac{1}{V}\frac{C_P}{\beta}\,dV + \kappa\,\frac{C_V}{\beta}\,dP$$

Thermodynamic Potentials:

$$G = U - TS + PV \qquad H = U + PV \qquad F = U - TS$$

Thermodynamic Identities:

(recall that each can be expanded in a fashion as shown in the first case for dU)

$$dU = T\,dS - P\,dV + \mu dN$$

$$= (\tfrac{\partial U}{\partial S}|_{(V,N)}\,dS + (\tfrac{\partial U}{\partial V}|_{(S,N)}\,dV + (\tfrac{\partial U}{\partial N}|_{(S,V)}\,dN$$

$$dH = T\,dS + V\,dP + \mu dN$$

$$dF = -P\,dV - S\,dT + \mu dN$$

$$dG = V\,dP - S\,dT + \mu dN \qquad \text{where } \mu \text{ is the chemical potential}$$

For an ideal gas, $\mu(T,P) = \mu^0(T) + kTln\left(\frac{P}{P^0}\right)$

Clausius-Clapeyron: $\frac{dP}{dT} = \frac{\Delta S}{\Delta V} = \frac{L}{T\,\Delta V}$ Joule-Kelvin coefficient: $\mu = (\frac{\partial T}{\partial P}|_H$

Maxwell Relations:

$(\frac{\partial T}{\partial V}|_S = -(\frac{\partial P}{\partial S}|_V \qquad (\frac{\partial T}{\partial P}|_S = (\frac{\partial V}{\partial S}|_P$

$(\frac{\partial S}{\partial V}|_T = (\frac{\partial P}{\partial T}|_V \qquad (\frac{\partial S}{\partial P}|_T = -(\frac{\partial V}{\partial T}|_P$

Engines/Refrigerators:

efficiency, e, or coefficient of performance, $c = benefit/cost$

Heat engine, $e = \frac{W}{Q_h}$, Rankine: $e = 1 - \frac{H_4 - H_1}{H_3 - H_1}$

Refrigerator, $c = \frac{Q_c}{W}$, Rankine: $c = \frac{H_1 - H_3}{H_2 - H_1}$, Heat pump: $c = \frac{Q_h}{W}$

Boltzmann factor: $e^{-E(s)/kT}$ Probability: $P(s) = \frac{1}{Z}\,e^{-E(s)/kT}$

Partition function: $Z = \Sigma_s\,e^{-E(s)/kT}$ $F = -kTlnZ$

$\overline{E} = -\frac{\partial(lnZ)}{\partial\beta}$ $\overline{E^2} = \frac{1}{Z}\frac{\partial Z^2}{\partial\beta^2}$ (V constant) $\beta = \frac{1}{kT}$

Maxwell-Boltzmann velocity distribution:

$N(v)dv \;=\; N\left(\frac{m}{2\pi kT}\right)^{3/2}\, 4\pi v^2 e^{-mv^2/(2kT)}\, dv$

$Z \;=\; \frac{Z_1^N}{N!}$ (ideal gas)

Partition function of ideal gas: $ln(Z) \;=\; N\,[ln(V) \,+\, ln(Z_{int}) \,-\, ln(N) \,-\, ln(v_Q) \,+\, 1]$

quantum volume: $v_Q \;=\; \left(\frac{h^2}{2\pi mkT}\right)^{3/2}$

Gibbs factor: $e^{-[E(s)-\mu N(s)]/kT}$

Grand partition function: $\mathcal{Z} \;=\; \Sigma_s\, e^{-[E(s)-\mu N(s)]/kT}$

$\overline{N} \;=\; \frac{kT}{\mathcal{Z}}\frac{\partial \mathcal{Z}}{\partial \mu} \qquad \overline{N^2} \;=\; \frac{(kT)^2}{\mathcal{Z}}\frac{\partial^2 \mathcal{Z}}{\partial \mu^2}$

$\overline{n} \;=\; \Sigma n P(n)$

$\overline{n} \;=\; \frac{1}{e^{(\epsilon-\mu)/kT}}$ (distinguishable particles)

$\overline{n} \;=\; \frac{1}{e^{(\epsilon-\mu)/kT}+1}$ Fermi energy, $\epsilon_F \;=\; \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}$ (Fermions)

$\overline{n} \;=\; \frac{1}{e^{(\epsilon-\mu)/kT}-1}$ (Bosons)

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