Start with general expression for heat capacity #15 on home page for Phy372

when $P = \text{constant} \Rightarrow dP = 0$

this is C_V

$$C_p = \left(\frac{\partial U}{\partial T}\Big|_V\right) + \left[\frac{\partial U}{\partial V}\Big|_T + P\right]\frac{dV}{dT} \qquad -(1)$$

expand $\rightarrow$ $$dV = \frac{\partial V}{\partial T}\Big|_P dT + \frac{\partial V}{\partial P}\Big|_T \cancel{dP^{\,0}} = \frac{\partial V}{\partial T}\Big|_P dT \Rightarrow \frac{dV}{dT} = \frac{\partial V}{\partial T}\Big|_P \qquad -(2)$$

rewrite eqn (1) $$C_p = \frac{\partial U}{\partial T}\Big|_V + \frac{\partial U}{\partial V}\Big|_T \frac{dV}{dT} + P\frac{dV}{dT} \qquad -(2.5)$$

expand $$dU = \frac{\partial U}{\partial T}\Big|_P dT + \frac{\partial U}{\partial P}\Big|_T \cancel{dP^{\,0}} \Rightarrow \frac{dU}{dT} = \frac{\partial U}{\partial T}\Big|_P \qquad -(3)$$

expand again $$dU = \frac{\partial U}{\partial T}\Big|_V dT + \frac{\partial U}{\partial V}\Big|_T dV$$

rearrange $\Rightarrow$ $$\frac{\partial U}{\partial T}\Big|_V = \frac{dU}{dT} - \frac{\partial U}{\partial V}\Big|_T \frac{dV}{dT}$$ put into 2.5

$$C_p = \left[\frac{dU}{dT} - \frac{\partial U}{\partial V}\Big|_T \cancel{\frac{dV}{dT}}\right] + \cancel{\frac{\partial U}{\partial V}\Big|_T \frac{dV}{dT}} + P\frac{dV}{dT}$$

$$= \frac{dU}{dT} + P\frac{dV}{dT}$$ But from (2) + (3) this is

$$C_p = \frac{\partial U}{\partial T}\Big|_P + P\frac{\partial V}{\partial T}\Big|_P$$ which is eqn 1.45 text