midterm solutions

#1)

c_p: intensive

P: intensive

T: intensive

v: intensive

ρ: intensive

#2)

$$\beta \equiv \frac{1}{v}\frac{\partial v}{\partial T}\Big|_P = \frac{R}{vP} \Rightarrow \frac{\partial v}{\partial T}\Big|_P = \frac{R}{P} \qquad -①$$

$$K \equiv \frac{-1}{v}\frac{\partial v}{\partial P}\Big|_T = \frac{RT}{vP^2} \Rightarrow \frac{\partial v}{\partial P}\Big|_T = \frac{-RT}{P^2} \qquad ②$$

Differentiate ① w.r.t P to find $-\frac{R}{P^2}$

Differentiate ② w.r.t. T to find $-\frac{R}{P^2}$

so yes there is a unique eq'n of state

#3

a) isothermal expansion:

$$P_i V_i = nRT_i \Rightarrow \underline{n} = \frac{(8)(1.01\times10^5)(4)}{(8.31)(400)} = 972.3 \text{ moles}$$

isothermal so $\underline{T_f} = T_i = 400k$

$$P_f V_f = nRT_f \Rightarrow V_f = \frac{(972.3)(8.31)(400)}{(1)(1.01\times10^5)} = 31.999 = 32m^3$$

$$W = -\int_{V_i}^{V_f} P\,dV = -\int \frac{nRT}{V}\,dV = -nRT\int \frac{dV}{V} = -nRT\ln\left(\frac{V_f}{V_i}\right)$$

$$= -(972.3)(8.31)(400)\ln\left(\frac{32}{4}\right) = -6.72\times10^6 \text{ J}$$

$\Delta U = Q + W$, $\Delta U = 0$ (isothermal)

so

$$Q = -W = +6.72\times10^6 \text{ J}$$

"monatomic" f=3

b) $n = 972.3$ moles (same)

$$\gamma = \frac{f+2}{f} = \frac{3+2}{3} = \frac{5}{3}$$

$$P_i V_i^{\gamma} = P_f V_f^{\gamma} \Rightarrow V_f = V_i\left(\frac{P_i}{P_f}\right)^{1/\gamma} = 4\left(\frac{8}{1}\right)^{3/5} = 13.9\,m^3$$

$$V_i T_i^{f/2} = V_f T_f^{f/2}$$

$$\text{so} \quad T_f = T_i\left(\frac{V_i}{V_f}\right)^{2/f} = (400)\left(\frac{4}{13.9}\right)^{2/3} = 174 \text{ K}$$

$$\Delta U = n\frac{f}{2}R(T_f - T_i) = \underset{0}{\cancel{Q}} + W \Rightarrow W = (972.3)\left(\frac{3}{2}\right)(8.31)[174-400]$$

$$W = -2.74\times10^6 \text{ J}$$

$$\Delta U = W = -2.74\times10^6 \text{ J}$$

$$Q = 0$$

c) In both cases W -ve <u>on</u> gas so gas is doing work on something external.