The "TdS" equations (solid, liquid, or gas)

Recall that we had $TdS = dU + PdV - \mu\, dN$
If we insist that $N = \text{const}$, then we can write

$$TdS = dU + PdV$$

We can manipulate and expand the various functions, rearranging etc (for example see the kind of thing we did in #9 and #14 on the web page) to find:

(#1) $$TdS = C_V\, dT + T \left.\frac{\partial P}{\partial T}\right|_V dV$$

(#2) $$TdS = C_P\, dT - T \left.\frac{\partial V}{\partial T}\right|_P dP$$

(#3) $$TdS = C_P \left.\frac{\partial T}{\partial V}\right|_P dV + C_V \left.\frac{\partial T}{\partial P}\right|_V dP$$

} all are equivalent; choose depending on what you know

If we continue to substitute:

(#1a) $$TdS = C_V\, dT + T\frac{\beta}{\kappa}\, dV$$

(#2a) $$TdS = C_P\, dT - \beta V T\, dP$$

(#3a) $$TdS = \frac{C_P}{\beta V}\, dV + \frac{\kappa C_V}{\beta}\, dP$$

} choose if β, κ const. over range of interest

where $\beta = \frac{1}{V}\left.\frac{\partial V}{\partial T}\right|_P$ $\quad \kappa = -\frac{1}{V}\left.\frac{\partial V}{\partial P}\right|_T$