Black Hole Formation From Stellar Core Collapse

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INTRODUCTION

In this report we go through the theory and simulations of the formation of stellar mass black holes (SmBH). We begin by giving a theoretical motivation for the plausibility of the formation of a black hole using simple toy models. We will then focus on the evolution of massive stars, with emphasis on the effect of mass loss, and the dynamics of iron core collapse. Inspired by the Russell Vogt theorem, we present the likely end points of massive star evolution and its dependence on metallicity from numerical simulations. We end by giving a likely mass distribution function for SmBHs.

THEORY

Schwarzschild's Solution

Following Einstein's development [1] of the field equations:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{1}$$

the following year Schwarzschild published [2] the solution for a point mass in a spherically symmetric, asymptotically flat vacuum. The resulting spacetime has the metric

$$ds^2 = -(1-2m/r)dt^2 + \frac{1}{1-2m/r}dr^2 + r^2 d\Omega_2^2 \tag{2}$$

where t is the time at infinity, r the radial coordinate, and $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\varphi^2$ is the metric of a 2-sphere. A metric provides a way of measuring distances on a spacetime; for example the metric of Euclidean space in Cartesian coordinates is simply the Pythagorean theorem. Note that Eq. 2 is written in geometrical units such that $c = G = 1$. The main features of this spacetime are:

1. There exists a singularity at $r = 0$, at which the curvature invariants all diverge to infinity.
2. There are no paths for photons to escape $r \leq 2m$.
3. All particles in the region $r \leq 2m$ will hit the singularity in finite proper time.
4. An observer with constant $r > 2m$ observing a particle as it descends through $r = 2m$ will see the particle get increasingly redshifted as $r \to 2m$, but will never observe it pass through. It would appear as a black hole.

The radius where $r = 2m$ is called the Schwarzschild radius. A star of a mass of M has a Schwarzschild radius, in SI units, of

$$r_{\text{Schw.}} = \frac{2GM}{c^2} = 2.95\text{km}\left(\frac{M}{M_{\text{Sun}}}\right) \tag{3}$$

For a neutron star with a mass of $M = 2M_{\text{Sun}}$, this corresponds to a Schwarzschild radius of about 6km. Given that the radius of such a neutron star is on the order of 10km, it is plausible that a black hole could form from the same or similar process that produces neutron stars, namely stellar core collapse.

Toy Models and Neutron Stars

Oppenheimer and Snyder show [3] that a homogeneous sphere of pressureless dust collapses homologously under its own gravity in a finite proper time to a singularity. It is clear however that a sphere of pressureless dust is a poor model for a star. A better model would be a sphere of perfect fluid which has an equation of state described [4] by the Tolman-Oppenheimer-Volkoff equation

$$\frac{dP}{dr} = -\frac{m\rho}{r^2}\left(1+\frac{P}{\rho}\right)\left(1+\frac{4\pi r^3 P}{m}\right)\left(1-\frac{2m}{r}\right)^{-1} \tag{4}$$

A fluid sphere with $R_* \leq r_{\text{Schw.}}$, would unavoidably collapse to a singularity in finite proper time. Buchdahl later shows [5] that for a fluid sphere with finite isotropic pressure and a non-increasing density, two very reasonable assumptions, the radius of the fluid sphere must satisfy the inequality

$$R_* \geq r_{\text{Buch.}} \equiv 4.33\text{km}\left(\frac{M}{M_{\text{Sun}}}\right) \tag{5}$$

If the inequality is not satisfied, the fluid sphere would requires infinite central pressure, meaning that the sphere would begin to collapse. Assuming the pressure maintains isotropy during collapse, the fluid would collapse to a singularity.

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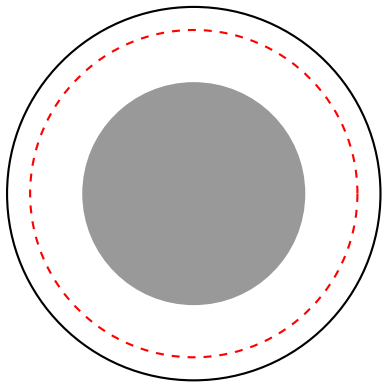

FIG. 1. Above is a diagram for a $M = 2M_{\text{Sun}}$ neutron star. The solid black line represents the surface of the neutron star ($R_* = 9.87$km), the red dashed line represents the Buchdahl bound for a perfect fluid sphere ($r_{\text{Buch.}} = 8.66$km) and the solid grey region represents the Schwarzschild radius ($r_{\text{Schw.}} = 5.90$km). We can see that it is plausible that a black hole could form in stellar core collapse scenarios similar to those that produce neutron stars.

For neutron stars, the dominant source of pressure is neutron degeneracy pressure, and is approximately:

$$P_{\text{n, deg}} \approx 9.65 \times 10^9 \left(\frac{M}{M_{\text{Sun}}}\right)^{5/3} \left(\frac{R_{\text{Sun}}}{R}\right)^5 \tag{6}$$

Since the radius of a neutron star is approximately given by:

$$R_* \approx 12.44 \text{ km} \left(\frac{M}{M_{\text{Sun}}}\right)^{-1/3} \tag{7}$$

Combining these results together we get that

$$P_{\text{n, deg}} \approx 5.26 \times 10^{33} \left(\frac{M}{M_{\text{Sun}}}\right)^{10/3} \tag{8}$$

For a $2M_{\text{Sun}}$ neutron star, this pressure is on the order of 5×10^{34}ergs cm^{-2}. Buchdahl's central pressure need not go all the way to infinity for collapse to occur, it just needs to approach the maximum neutron degeneracy pressure.

Finding exact solutions to the field equations for more realistic collapse scenarios are infamously difficult to find. The collapse described by Oppenheimer and Snyder, and the condition described by Buchdahl both require spherical symmetry (meaning no rotation), and no cosmological constant. In more realistic models of collapse, numerical simulations are required.

HIGH MASS STARS

Mass Loss Rates

For a wide range of masses, star luminosity is approximately $L \propto M^3$. Stars with 100 times the mass of the sun are about a million times more luminous. The vast amounts of radiation near the stellar surface transfer momentum to the gas by absorption and scattering by UV metal lines, particularly Si IV, C IV, N V, and S VI [6]. The mass loss rates resulting from the stellar winds is therefore dependent on the luminosity, metallicity, and the ionization (which is dependent on temperature).

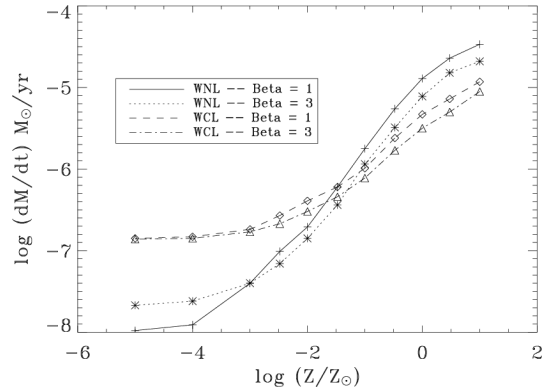

FIG. 2. Mass loss rate to metallicity relation for WN and WC stars (WR stars for which N and C spectra dominate). β is a power in the model of the stellar wind velocity, $v(r) = v_{\text{term}}(1 - R/r)^\beta$

It is important to develop accurate models of mass loss for massive stars, since this affects the evolution on the HR diagram, and may change the type of compact object that remains after core collapse. The simulations by Heger et al. [7] use

$$\dot{M} \propto -\sqrt{Z} \tag{9}$$

for hot main sequence stars, red supergiants (RSG) and Wolf Rayet (WR) stars. The mass loss for WR stars is sensitive to the presence of Fe, and requires modeling stellar wind clumping. Models of mass loss rates are given in FIG. 2, and to first order, this agrees with equation 9.

Iron Core Collapse

Main sequence stars fuse hydrogen in their cores into helium for most of their lives. When the hydrogen in the core is used up, the star will initially contract, heat up and helium burning will begin. For very massive stars this process continues through helium, carbon, oxygen and so on, until there is a degenerate iron core, held up by thermal pressure and electron degeneracy pressure. The timeline for the life of a typical $25M_{\text{Sun}}$ star is:

Stage	Duration
H $\rightarrow$ He	7×10^6 years
He $\rightarrow$ C	7×10^5 years
C $\rightarrow$ O	600 years
O $\rightarrow$ Si	6 months
Si $\rightarrow$ Fe	1 day
Fe core collapse	0.25 seconds

Iron nuclei cannot fuse into other nuclei exothermically, so this process terminates. At the onset of core collapse, the temperature is high enough that the iron dissociates via photodisintegration into alpha particles,

$$^{56}_{26}\mathrm{Fe} + \gamma \rightarrow 13\,^{4}_{2}\mathrm{He} + 4n \tag{10}$$

$$^{4}_{2}\mathrm{He} + \gamma \rightarrow 2p^{+} + 2n \tag{11}$$

which is an endothermic process and the thermal pressure drops. The compression of the core leads to electron capture by the protons,

$$p^{+} + e^{-} \rightarrow n + \nu_e \tag{12}$$

reducing the electron degeneracy pressure. The further collapse accelerates the rate of the iron nuclei dissociating and collapse continues further. At some point the core becomes a proto-neutron star, with a radius of approximately 30km, and is held up by neutron degeneracy pressure.

The outer layers of the star, in particular the silicon burning layer, fall towards the collapsed core. The shock wave that radiates out from the collapse encounters the infalling matter and stalls. This is the critical moment of the collapse. It is believed that the neutrino luminosity due to the electron capture in the core is responsible for powering the stalled shock. In order to have sufficient neutrino flux to power the shock, there must be a continuous accretion of matter onto the the proto-neutron star. A 100 to 300km radius region below the stalled shock will form, which will have a large temperature gradient due to the neutrino heating its lower layers, and will be convectively unstable. Numerical simulations [8] [9] of this convective region, show rising pockets of high entropy matter along with descending filaments of silicon accretion onto the core.

If the accretion is continuous and at a high enough rate, the shock will have sufficient energy to expel the outer layers of the star. If the explosion is weakened due to a lack of accretion, the shock wave will expel some of the infalling matter but enough will remain to smother the core and a black hole will form. If the convective region is very weak, it cannot power the shock, and infalling matter overwhelms the core and directly forms a black hole.

Fig. 2. from Modeling CoreCollapse Supernovae in Three Dimensions
Fryer & Warren 2002 ApJL 574 L65 doi:10.1086/342258
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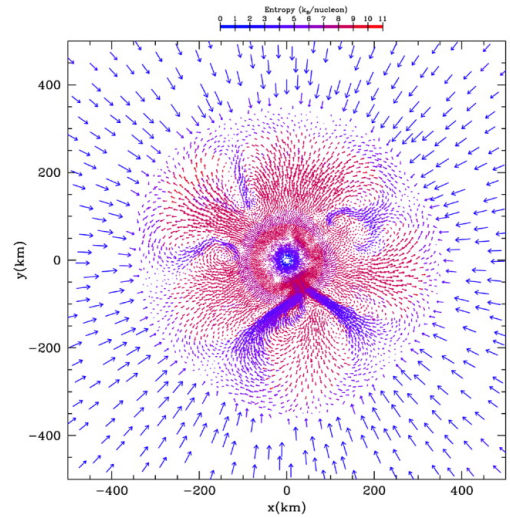

FIG. 3. Convective region around proto-neutron star, during core collapse. Velocity vectors coloured according to entropy. The cold, low-entropy material flows downward as the high-entropy material bubbles upward.

ENDPOINTS OF STELLAR EVOLUTION

Figures FIG. 4 and FIG. 5 are the result of numerical simulations by Heger et al. [7], who determined the evolution of massive stars $9M_{\mathrm{Sun}} \leq M \leq 300M_{\mathrm{Sun}}$, without rotation from ZAMS to death as either iron core collapse supernovae or pair instability supernovae. The type of stellar remnant is determined largely by the carbon oxygen core mass, and the type of supernova is determined by the the presence of a hydrogen envelope or lack thereof; supernovae with hydrogen spectra are Type II, and without hydrogen spectra is Type I. The subtype of Type II supernovae is determined by the hydrogen envelope mass, whereas the subtype of Type I supernovae is determined by the helium core mass.

We can see in FIG. 4 from the results of the simulation that the minimum ZAMS mass required for an iron core to collapse and form a neutron star is around $9M_{\mathrm{Sun}}$. Around $25M_{\mathrm{Sun}}$, the shock of the core collapse does not gain sufficient power through the neutrino heating to fully eject the outer layers of the star, and some mass falls back onto the proto-neutron star, forming a black hole. Around 40 stellar masses, the the shock is overcome by the infalling matter, and the collapse directly forms a black hole. The bright green line represents the boundary where the hydrogen envelope is removed from the star by strong stellar winds. To the right of this

Fig. 1. from How Massive Single Stars End Their Life
Heger et al. 2003 ApJ 591 288 doi:10.1086/375341
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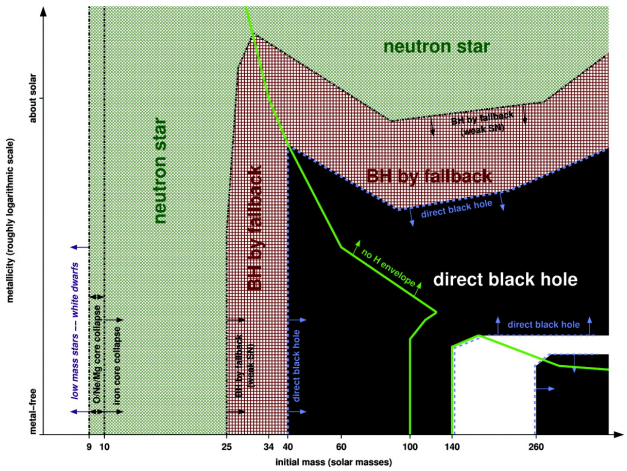

FIG. 4. Stellar remnant determined by initial mass and initial bulk metallicity. The white region on the far left results in a white dwarf remnant, the green region a neutron star, red hatched region results in a black hole by fallback due to a weak supernova, and black region forms a black hole directly due to very weak explosion.

Fig. 2. from How Massive Single Stars End Their Life
Heger et al. 2003 ApJ 591 288 doi:10.1086/375341
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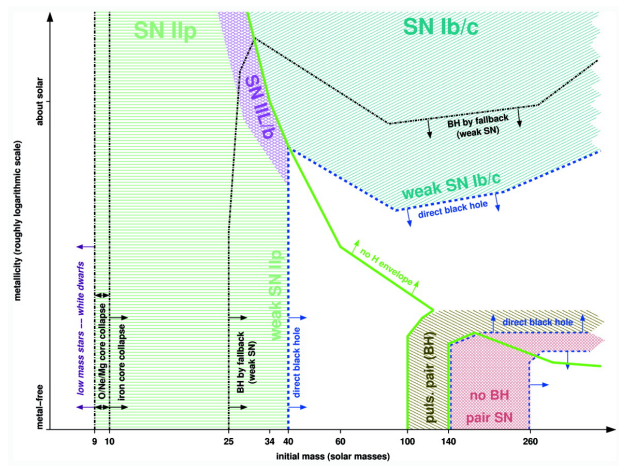

FIG. 5. Supernova type determined by initial mass and bulk metallicity. Type II occurs when there is a hydrogen envelope, and Type I occurs in its absence. Direct to black hole collapse had a very weak explosion.

curve, the star is effectively a helium star at time of core collapse. As metallicity increases, so does opacity, resulting in increased mass loss. In this simulation, a $100M_{\rm Sun}$ ZAMS star with solar metallicity will lose enough matter in its lifetime by stellar winds the core collapse shock is able eject the remaining outer layers of the star, resulting in a neutron star.

We can see in FIG. 5 the classification of the resulting supernova. For masses $9M_{\rm Sun} < M < 40M_{\rm Sun}$ and low metallicities, hydrogen spectra is produced from the hydrogen envelope. In the mass range where a black hole is formed by infall, the resulting supernova is weak. The p in SN IIp comes from the characteristic plateau shape of the decaying luminosity curve. The shock wave ionises the hydrogen envelope, resulting in a reduction of opacity, delaying the decay of the light curve. SNIIL/b supernovae initially have a weak hydrogen line and as the outer layers expands, the hydrogen envelope lowers its opacity revealing the helium star underneath. When a black hole is formed directly, the shock was overcome by infalling matter, so no supernova is produced at all.

For high metallicities, the lack of a hydrogen envelope is responsible for the missing hydrogen spectra, meaning the supernova will be classified as SNIb or SNIc.

BLACK HOLE MASSES

The stellar mass black hole that remains from an iron core collapse can also be simulated. Although Heger et al. [7] does not provide an initial mass function for black holes from their simulations, similar work by Belczynski et al. [10] and Fryer and Kalogera [11] does produce a remnant mass profile.

The simulations by Belczynski report similar findings to Heger: initial star masses below 8 solar masses result in a white dwarf, between 8 and 20 solar masses result in a neutron star, and above 20, a black hole. FIG. 6 shows that black hole formed from high mass stars are usually between 10 to 20 solar masses. It appears that when a black hole forms by infall, the final mass can be higher by a factor of 2. Also for increasing metallicity, the black hole mass is suppressed at the higher mass range, reaching a maximum of about 10 solar masses.

Fryer uses assumptions about the initial mass function for ZAMS stars, and the fraction of the total explosion energy used to unbind the outer layers of the star, and produces a remnant mass distribution function (shown in FIG. 7) Using single power law for the initial mass function of the progenitor stars:

$$dF \propto M^{-\gamma} dM \tag{13}$$

he finds that about 30% of the remnants are black holes (for $\gamma = 2$) and about 12% are black holes for ($\gamma = 3$), assuming all objects below $3M_{\rm Sun}$ are neutron stars.

Fig. 1. from A Comprehensive Study of Young Black Hole Populations
Belczynski, Sadowski, & Rasio 2004 ApJ 611 1068 doi:10.1086/422191
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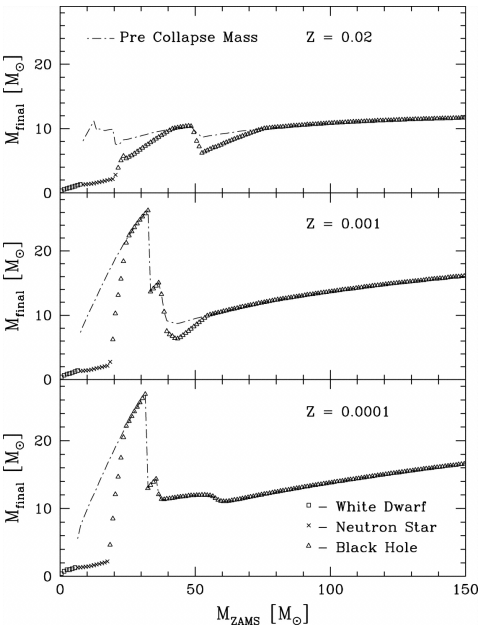

FIG. 6. Initial to final mass relation for different metallicites. Remnants of different types (WDs, NSs, and BHs) are marked.

Fig. 3. from Theoretical Black Hole Mass Distributions
Fryer & Kalogera 2001 ApJ 554 548 doi:10.1086/321359
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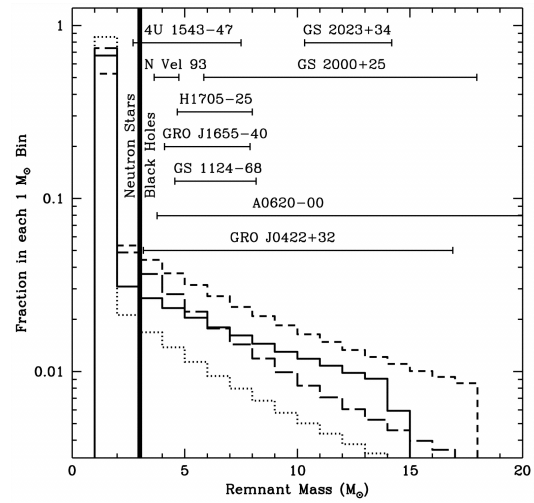

FIG. 7. The mass distribution of core collapse renmants. For $f = 1$ (solid and dotted lines) and $f = 0.1$ (short- and long-dashed lines). Initial mass function with power $\gamma = 2$ (solid and short-dashed lines) and $\gamma = 3$ (dotted and long-dashed lines)

CONCLUSION

In this report we have relied heavily on simulations in the literature to assert likely iron core collapse outcomes, and the mass distributions of the compact remnants. It should be clear however that this is entirely model dependent, and one should keep in mind the uncertainties in the stellar winds, mass loss rates, and models of neutrino heating and convection.

[1] A. Einstein. Die Feldgleichungen der Gravitation. *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin), Seite 844-847.*, 1915.

[2] K. Schwarzschild. On the Gravitational Field of a Mass Point According to Einstein's Theory. *Abh. Konigl. Preuss. Akad. Wissenschaften Jahre 1906,92, Berlin,1907*, 1916, 1916.

[3] J. R. Oppenheimer and H. Snyder. On continued gravitational contraction. *Phys. Rev.*, 56:455–459, Sep 1939.

[4] J. R. Oppenheimer and G. M. Volkoff. On massive neutron cores. *Phys. Rev.*, 55:374–381, Feb 1939.

[5] H. A. Buchdahl. General relativistic fluid spheres. *Phys. Rev.*, 116:1027–1034, Nov 1959.

[6] L. B. Lucy and P. M. Solomon. Mass Loss by Hot Stars. *Astrophys. J.*, 159:879, March 1970.

[7] A. Heger, C. L. Fryer, S. E. Woosley, N. Langer, and D. H. Hartmann. How massive single stars end their life. *The Astrophysical Journal*, 591(1):288, 2003.

[8] Chris L. Fryer and Michael S. Warren. Modeling Core-Collapse Supernovae in Three Dimensions. *Astrophys. J.*, 574:L65–L68, July 2002.

[9] A. Mezzacappa, S. W. Bruenn, E. J. Lentz, W. R. Hix, O. E. B. Messer, J. A. Harris, E. J. Lingerfelt, E. Endeve, K. N. Yakunin, J. M. Blondin, and P. Marronetti. Two- and Three-Dimensional Multi-Physics Simulations of Core Collapse Supernovae: A Brief Status Report and Summary of Results from the "Oak Ridge" Group. In N. V. Pogorelov, E. Audit, and G. P. Zank, editors, *8th International Conference of Numerical Modeling of Space Plasma Flows (ASTRONUM 2013)*, volume 488, page 102, September 2014.

[10] Krzysztof Belczynski, Aleksander Sadowski, and Frederic A. Rasio. A Comprehensive Study of Young Black Hole Populations. *Astrophys. J.*, 611:1068–1079, August 2004.

[11] Chris L. Fryer and Vassiliki Kalogera. Theoretical Black Hole Mass Distributions. *Astrophys. J.*, 554:548–560, June 2001.