Multiplicity of a Large Einstein solid
q, N, both large, N = # oscillators

In the High Temperature limit i.e. $q \gg N$, $\frac{N}{q} \ll 1$

We had $\Omega(N,q) = \frac{(q+N-1)!}{q!(N-1)!} \approx \frac{(q+N)!}{q!N!}$

then $\ln \Omega = \ln[(q+N)!] - \ln(q!) - \ln(N!)$

Now use Stirling's law which says $\ln(x!) = x\ln x - x$

so $\ln \Omega = [(q+N)\ln(q+N) - (q+N)] - [q\ln q - q] - [N\ln N - N]$

$= (q+N)\ln(q+N) - q\ln q - N\ln N$

$= (q+N)\ln\left[q\left(1+\frac{N}{q}\right)\right] - q\ln q - N\ln N$

recall that $\ln(1+x) \approx x$ when $|x| \ll 1$ so

$\ln \Omega = (q+N)\left(\ln q + \frac{N}{q}\right) - q\ln q - N\ln N$

$= \left[q\ln q + N\ln q + N + \frac{N^2}{q}\right] - q\ln q - N\ln N$

$= N\ln q + N + \frac{N^2}{q} - N\ln N = N\ln\left(\frac{q}{N}\right) + N\left(1+\frac{N}{q}\right)$

$= N\ln\left(\frac{q}{N}\right) + N$

$\therefore \ln \Omega = N\left[\ln\left(\frac{q}{N}\right) + 1\right]$ to get entropy: $S = k\ln(\Omega)$

what is Ω? $\Omega = \left\{e^1\left[e^{\ln(q/N)}\right]\right\}^N = \left(e\frac{q}{N}\right)^N$ for $q \gg N$ eq'n 2.21 text high T limit

In summary:

$\Omega = \frac{(q+N-1)!}{q!(N-1)!} \approx \left(\frac{eq}{N}\right)^N$ see eq'n of Prob. 2.18 of text for general solution