PHYSICS 372 In-Class Test

February 15, 2018; 8:30 - 9:20 a.m.

1. Specify whether the following quantities are extensive or intensive: specific heat capacity (c_P), pressure (P), temperature (T), molar-specific volume (v), density (ρ).

2. The following response functions are found for a certain substance:

$$\beta = \frac{R}{v\,P} \qquad\qquad \kappa = \frac{R\,T}{v\,P^2} \tag{1}$$

Determine whether or not a unique equation of state can be found.

3. An ideal monatomic gas containing n moles that has initial values of pressure, $P_i = 8$ atm, volume, $V_i = 4$ m^3, and temperature, $T_i = 400$ K, is allowed to expand until the final pressure is $P_f = 1$ atm.

(a) If the expansion is *isothermal*, find the following quantities:

$$n,\ V_f,\ T_f,\ W,\ Q,\ \text{and}\ \Delta\,U$$

where the subscripts, f mean 'final' and the last 3 quantities refer to the work done on the gas, the heat into or out of the gas, and the change in internal energy of the gas, respectively. Make sure that you specify the units of your answer.

(b) Repeat part (a) for but *adiabatic* expansion.

(c) For both cases, is work being done on the expanding gas or is the expanding gas doing work on something external?

PHYSICS 372: Data and Equations

Mass of an atomic mass unit	$u \approx m_p = 1.67 \times 10^{-27}$ kg
Planck's constant	h = 6.626×10^{-34} J s
Boltzmann's constant	k = 1.38×10^{-23} J K^{-1}
Avogadro's number	$N_A = 6.02 \times 10^{23}$
Gas constant	R = 8.315 J/(mol K)
Energy of 1 eV	= 1.60×10^{-19} J

T (in °C) = T(in K) - 273

1 atm = 1.013 bar = 1.013×10^5 N/m^2 1 cal = 4.186 J

$$f = f(x,y) \implies df = \frac{\partial f}{\partial x}\Big|_y dx + \frac{\partial f}{\partial y}\Big|_x dy$$

Stirling's Approx: $N! \approx N^N e^{-N}\sqrt{2\pi N}$ $\quad ln(N!) \approx NlnN - N \quad$ (N large)

$ln(1+x) \approx x \qquad (1+ax)^{1/x} \approx e^a \qquad (|x| << 1)$

Cyclic relation for P, V, & T: $\left(\frac{\partial V}{\partial P}\Big|_T\right.\left(\frac{\partial P}{\partial T}\Big|_V\right.\left(\frac{\partial T}{\partial V}\Big|_P\right. = -1$

Ideal gas law: $PV = NkT = nRT$

Ideal gas rms speed: $v_{rms} = \sqrt{\frac{3kT}{\mu m_p}}$ where $\mu \equiv \frac{<m>}{m_p}$

Van der Waal's gas: $\left(P + \frac{a}{v^2}\right)(v - b) = RT$ where $v = V/n$

$\gamma = \frac{f+2}{f}$ Quadratic degrees of freedom: $U = N\frac{f}{2}kT$

Volume expansion coefficient: $\beta \equiv \frac{1}{V}\left(\frac{\partial V}{\partial T}\Big|_P\right.$ Isothermal compressability: $\kappa \equiv -\frac{1}{V}\left(\frac{\partial V}{\partial P}\Big|_T\right.$

Sound speed: $c_s = \sqrt{\frac{B_s}{\rho}}$ where $\frac{B_s}{\gamma} = B = \frac{1}{\kappa} = \frac{1}{\gamma\,\kappa_s}$ (subscripts, s, meaning adiabatic)

First law: $\Delta U = Q + W \qquad dU = dQ + dW$

Expansive/compressive work: $dW = -P\,dV$

Ideal gas adiabatic expansion/compression: $V\,T^{f/2} = const \qquad P\,V^{\gamma} = const$

Enthalpy: $H = U + PV$ Latent heat: $L = \frac{Q}{m}$

Heat Capacities: $C_V \equiv \left(\frac{dQ}{dT}\Big|_V\right. = \left(\frac{\partial U}{\partial T}\Big|_V\right.$

$C_P \equiv \left(\frac{dQ}{dT}\Big|_P\right. = \left(\frac{\partial H}{\partial T}\Big|_P\right. = C_V + \left[\left(\frac{dU}{dV}\Big|_T\right. + P\right]V\beta$

Two-state system: $\Omega(N,n) = \frac{N!}{n!(N-n)!} \qquad (N > n)$

Einstein solid: $\Omega(N,q) = \frac{(q+N-1)!}{q!(N-1)!}$ (N the # of oscillators, $q\,\epsilon = U$)

$$\Omega = \left(\frac{e\,N}{q}\right)^q \quad (q << N), \qquad \Omega = \left(\frac{e\,q}{N}\right)^N \quad (q >> N)$$

Monatomic ideal gas: $\Omega = f(N)\,V^N\,U^{3N/2}$ Entropy: $S = k\,ln\Omega$

Sackur-Tetrode Eqn: $S = Nk\left\{ln\left[\frac{V}{N}\left(\frac{4\pi mU}{3Nh^2}\right)^{3/2}\right] + \frac{5}{2}\right\}$

$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\Big|_{N,V}\right. \qquad dS = \frac{dQ}{T} \qquad \Delta S = \int \frac{C_V}{T}\,dT = \int \frac{C_P}{T}\,dT$