midterm 2019

(#1) For ^{12}C, 1 mole weighs 12 g
so 1 g of ^{12}C is $1/12$ of a mole

then $U = n\frac{f}{2}RT$ (quadratic deg of freedom)

$$= \left(\frac{1}{12}\right)\left(\frac{6}{2}\right)(8.315)(298) = 619\ \text{J}$$

(#2) EoS is: $P = \frac{1}{v}\left(RT - \frac{a}{bT^{1/2}}\right)$

$$\kappa_T \equiv \frac{-1}{v}\frac{\partial v}{\partial P}\bigg|_T = -\frac{1}{v}\frac{1}{\frac{\partial P}{\partial v}\big|_T} \qquad \frac{\partial P}{\partial v}\bigg|_T = \frac{-1}{v^2}\left(RT - \frac{a}{bT^{1/2}}\right)$$

$$\text{so } \kappa_T = -\frac{1}{v}\frac{(-v^2)}{\left(RT - \frac{a}{bT^{1/2}}\right)} = \frac{v}{\left(RT - \frac{a}{bT^{1/2}}\right)} = \frac{1}{P}$$ same as ideal gas

$$\beta \equiv \frac{1}{v}\frac{\partial v}{\partial T}\bigg|_P$$ so rewrite EoS as $v = \frac{1}{P}\left(RT - \frac{a}{bT^{1/2}}\right)$

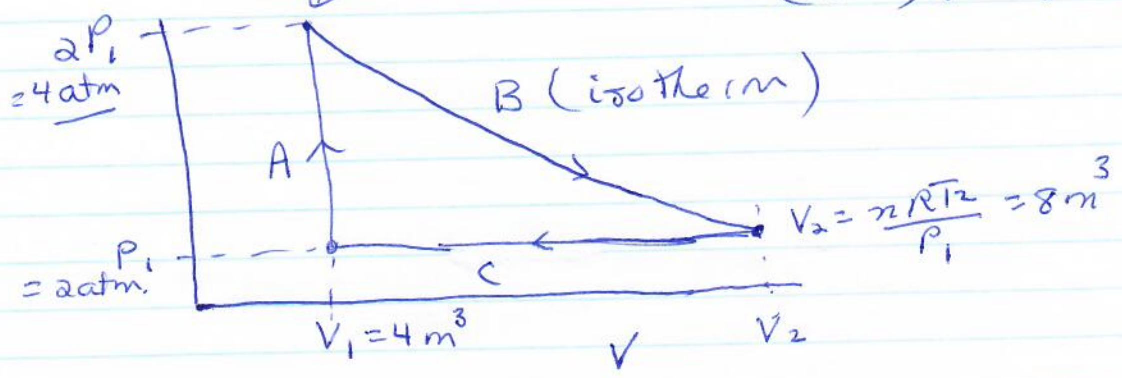
$$\text{then } \beta = \frac{1}{v}\left[\frac{1}{P}\left(R + \frac{1}{2}\frac{a}{bT^{3/2}}\right)\right] = \frac{1}{v}\left[\frac{v}{\left(RT - \frac{a}{bT^{1/2}}\right)}\right]\left(R + \frac{1}{2}\frac{a}{bT^{3/2}}\right)$$

get rid of P

$$= \frac{R + \frac{a}{2bT^{3/2}}}{RT - \frac{a}{bT^{1/2}}} = \frac{1}{T}\left(\frac{R + \frac{a}{2bT^{3/2}}}{R - \frac{a}{bT^{3/2}}}\right) = \frac{1}{T}\left(\frac{1 + \frac{a}{2bRT^{3/2}}}{1 - \frac{a}{bRT^{3/2}}}\right)$$

$2P_1V_1 = nRT_2 \Rightarrow T_2 = \frac{(4)(1.01\times10^5)(4)}{(2\times10^3)(8.315)} = 97.3\,K$

#3



a) $W_A = -\int P\,dV = 0$ because $dV = 0$

$W_B = -\int P\,dV = -nRT\int_{V_i}^{V_f}\frac{dV}{V} = -nRT\ln\frac{V_1}{V_2}$ find V_2 see graph

isotherm so T = const

$= -(2\times10^3)(8.315)(97.3)\ln\left(\frac{8}{4}\right) = -1.1\times10^6\,J$

$W_C = -\int P\,dV = -P_1(V_f - V_i) = -(2)(1.01\times10^5)(4-8) = +8.1\times10^5\,J$

$W_{net} = W_A + W_B + W_C = -3.0\times10^5\,J$

b) negative work is being done on the gas so the gas is doing +ve work on something external