15.1 The isothermal compressibility of an object is defined as

$$\kappa_T \equiv -\frac{1}{v}\frac{\partial v}{\partial P}\Big|_T \qquad (15.1)$$

where v is the *specific volume* (cm^3/g) and P is the pressure.
(a) Rewrite κ_T in terms of the density derivative, $\frac{\partial \rho}{\partial P}$.
(b) Find κ_T for a non-relativistic and relativistic C/O white dwarf.
(c) Evaluate κ_T for a typical non-relativistic and relativistic C/O white dwarf. Look up κ_T for the hardest substance on Earth and compare.

16.1 Although the pressure of a white dwarf is produced by electron degeneracy, the *internal energy*, U, depends on the ion temperature according to $U = C_1\,T_{int}$, where T_{int} is the internal (isothermal) temperature and C_1 is a constant. The luminosity of a white dwarf can be related to its internal temperature via $L = C_2\,{T_{int}}^{7/2}$, where C_2 is another constant[1].
(a) Derive an expression for the age, t, of a white dwarf as a function of its initial interior temperature, T_{int0} and its current interior temperature, T_{int}. Let $t = 0$ when $T = T_{int0}$ and note that $dU/dt = -L$.
(b) Reasonable values of the constants for a $1\ M_\odot$ white dwarf are $C_1 = 2 \times 10^{40}$ and $C_2 = 9.3 \times 10^5$ (cgs). If a $1\ M_\odot$ white dwarf starts with an interior temperature of 10^7 K, what would be its interior temperature after 13.7 billion years? Comment on the length of time it would take for a white dwarf to cool to a 'cinder'.

18.1 *(a)* Plot the $P\dot{P}$ diagram over the same ranges as shown in Fig. 18.10 using updated data (and the plotting facility) from the Australia Telescope National Facility (ATNF) Pulsar Catalogue (see
`https://www.atnf.csiro.au/research/pulsar/psrcat/`).
(b) Look up P and $\dot{P}$ for the *Vela Pulsar* ($M = 1.5\ M_\odot$).
(c) In class, I indicated that the mass-radius relation for a neutron star could be expressed similarly to a white dwarf, i.e. $R \propto M^{-1/3}$ and this relation is *roughly* what is seen in a handout in which various EOS are plotted. Find the dependence of the moment of inertia, I, on mass. If the pulsar mass increased by a factor of two, by what factor would I change?
(d) Assume that the moment of intertia is the same as that of the canonical pulsar given in the handout and compute the following quantities:
(i) the radius (km), *(ii)* the perpendicular magnetic field, $B_\perp$ (Gauss), and *(iii)* the average density (g cm^{-3}). Verify that the density falls within the range for neutron degeneracy pressure given in the handout table called "Composition of Neutron Star Material".
(e) Compute the characteristic age (yr) of the Vela Pulsar.